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ON THE ORBITAL STABILITY OF A PERIODIC SOLUTION OF THE EQUATIONS OF MOTION OF A KOVALEVSKAYA GYROSCOPE*

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The sufficient conditions for the orbital stability of a periodic solution of the equations of motion of a Kovalevskaya gyroscope in the case of Bobylev-Steklov integrability are obtained.

It is difficult to expect Lyapunov stability for the unsteady motions of a heavy solid having a fixed point since a dependence of the vibrations frequency on the initial conditions is characteristic for the simplest of them, i.e. periodic motions /1/. Moreover, a rougher property of periodic solutions of the Euler-Poisson equations, orbital stability /2/, is not the subject of special investigations in the dynamics of a solid. The algorithm of the present investigation utilizes the treatment ascribed Zhukovskii /3/ of orbital stability as the Lyapunov stability of motion for a special selection of the variable playing the part of time (see /4/ also) and the Chetayev method /5/ of constructing Lyapunov functions from the first integrals of the equations of perturbed motion. This latter circumstance enables the Chetayev method to be put in one series with the methods used in /1, 4, 6-9/, etc.

1. Under the Kovalevskaya conditions the Euler-Poisson equations and the first integrals have the following form in dimensionless variables /10/

$$2p^{\cdot} = qr, \quad 2q^{\cdot} = -rp - \gamma'', \quad r^{\cdot} = \gamma' \tag{1.1}$$
$$\gamma^{\cdot} = \gamma' r - \gamma'' q, \quad \gamma'^{\cdot} = \gamma'' p - \gamma r, \quad \gamma''^{\cdot} = \gamma q - \gamma' p$$

$$2(p^2 + q^2) + r^2 - 2\gamma = 6l_1, \quad 2(p\gamma + q\gamma') + r\gamma'' = 2l \tag{1.2}$$
$$\gamma^2 + \gamma'^2 + \gamma''^2 = 1, \quad (p^2 - q^2 + \gamma)^2 + (2pq + \gamma')^2 = k^2$$

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If the integration constants k, l_1, l are related by $l = (k^2 - p_0^4 - 1) / (2p_0)$, $l_1 = (3p_0^4 + 1 - k^2) / (6p_0^2)$, where $p_0 \neq 0$ is a constant whose mechanical meaning is indicated below, then (1.1) allows of a family of periodic solutions corresponding to the case of Bobylev-Steklov integrability /11/

$$\begin{aligned} &p = p_0,\ q = 0,\ r = r_0(t) = [2k\cos\varphi(t) + p_0^{-2}(1 - k^2 - p_0^4)]^{1/2}, \\ &\gamma = \gamma_0(t) = k\cos\varphi(t) - p_0^2 \\ &\gamma' = \gamma_0'(t) = -k\sin\varphi(t),\quad \gamma'' = \gamma_0''(t) = -p_0 r_0(t) \end{aligned} \tag{1.3}$$

The elliptic function $\varphi(t)$ satisfies the equation

$$\varphi^{\cdot} = r_0(t) \tag{1.4}$$

The solutions determined by (1.3) and (1.4) depend in a substantial manner on the arbitrary constants p_0 and k and separate out closed orbits /2/ denoted by $M(k, p_0)$ in the six-dimensional phase space of system (1.1). Without loss of generality, we shall henceforth assume $k \geqslant 0$.

We shall confine ourselves to studying the periodic solutions (1.3) and (1.4) in which the third component of the angular velocity $r_0(t) \neq 0$ during all the motion. The condition mentioned is satisfied if and only if

$$k + p_0^2 < 1 \tag{1.5}$$

When condition (1.5) is satisfied it follows from (1.4) that $\varphi(t)$ increases monotonically in the interval $[\varphi(t_0); +\infty[$. Because of the autonomy of system (1.1), we later set $t_0 = \varphi(t_0) = 0$ everywhere.

2. The configuration of the orbit $M(k, p_0)$ in the space R^6 is determined merely by the relationships (1.3), where φ can be considered to be a parameter. The law of motion of the mapping point over these orbits is given by (1.4).

We will examine arbitrarily close mapping points P_1 and P_2 moving over the orbits $M(k, p_0)$ and $M(k + \delta k,\ p_0 + \delta p_0)$ respectively, at the initial time. The modulus of the elliptic function $\varphi(t)$ depends on k and p_0; consequently, the periods of the motion over $M(k, p_0)$ and $M(k + \delta k,\ p_0 + \delta p_0)$ are distinct, and after a certain time the spacing between P_1 and P_2 will certainly surpass a certain previously assigned quantity. Therefore, solution (1.3), (1.4) is Lyapunov unstable for any p_0 and k satisfying inequality (1.5).

Before investigating the orbital stability of (1.3), (1.4), we will describe a certain general approach to the study of this property of the periodic solutions of autonomous systems.

We fix $x_0 \in R^n$ and we introduce the notation

$$I = [0; +\infty[,\quad S_\alpha = \{\xi \in R^n : \|\xi - x_0\| < \alpha\}$$

For all possible $\xi \in S_\alpha$ we consider the solutions

$$x = \eta(t, \xi),\quad \eta(0, \xi) = \xi \tag{2.1}$$

of the autonomous system

$$x^{\cdot} = f(x) \tag{2.2}$$

The mapping f possesses smoothness ensuring the existence for $t \in I$, $\xi \in S_\alpha$ and uniqueness of the solution of (2.2).

We assume that system (2.2) has the periodic solution

$$x = \eta(t, x_0) \tag{2.3}$$

whose orbit we denote by M.

Definition. The functional $\Pi : I \times S_\alpha \to I$ yields a stable parametrization, with respect to (2.3), of a set of solutions (2.1) if for each $\varepsilon > 0$ there is a $\delta > 0$ such that for any $\xi \in S_\delta$ the following conditions are satisfied:

1) The mapping $\tau \to t = \Pi(\tau, \xi)$ is a homeomorphism of I in I;
2) For all $\tau \in I$ the following inequality holds

$$\|\eta(\Pi(\tau, \xi), \xi) - \eta(\Pi(\tau, x_0), x_0)\| < \varepsilon$$

Let the set of solutions (2.1) allow of a stable parametrization, with respect to (2.3) of Π. For fixed ξ the functional Π possesses properties following from its definition

$$\Pi(0, \xi) = 0,\quad \lim_{\tau \to +\infty} \Pi(\tau, \xi) = +\infty$$

At the each time $\tau \in I$ the point $\eta(\Pi(\tau, \xi), \xi)$ of the perturbed trajectory is close to the point $\eta(\Pi(\tau, x_0), x_0)$ of the orbit M. This correspondence between the perturbed and unperturbed solution does not imply the stability of the latter in the Lyapunov sense since in general $\Pi(\tau, \xi) \neq \Pi(\tau, x_0)$. However, the presence of a stable parametrization, with respect to (2.3) of the set (2.1) is evidently a sufficient condition for the orbital stability of the periodic solution (2.3).

Let us mention a certain algorithm to confirm the existence of stable parametrization without using the explicit form of the solution (2.1).

To do this, we append an equation

$$\tau^{\cdot} = 1/g(x) \tag{2.4}$$

and the initial condition $\tau(0) = 0$ to (2.2). The function g here is continuously differentiable in the neighbourhood of the orbit M and for $\tau \in I$ satisfies the inequality (β is a constant)

$$g(\eta(t, x_0)) \geqslant \beta > 0 \tag{2.5}$$

To study the behaviour of the integral curves of system (2.2) in the neighbourhood of the orbit M we introduce a new independent variable τ. We obtain the equation

$$dx/d\tau = f(x)\, g(x) \tag{2.6}$$

The dependence of the variable τ on time in the periodic solution (2.3) is as follows:

$$\tau = \int_0^t \frac{d\theta}{g(\eta(\theta, x_0))} \tag{2.7}$$

By virtue of inequality (2.5) the functional (2.7) maps I homeomorphically into I. If Π denotes the inverse mapping to (2.7) $\tau \to t = \Pi(\tau, x_0)$, then system (2.6) will allow of the periodic solution

$$x(\tau) = \eta(\Pi(\tau, x_0), x_0) \tag{2.8}$$

whose orbit agrees with M.

Theorem 1. If the solution (2.8) of system (2.6) is Lyapunov stable, then the set (2.1) allows a stable parametrization with respect to the solution (2.3).

Proof. Let $\eta_*(\tau, \xi)$ denote the solution of system (2.6) with the initial data $\eta_*(0, \xi) = \xi$. Here

$$\eta(\Pi(\tau, x_0), x_0) = \eta_*(\tau, x_0)$$

The Lyapunov stability of the solution (2.8) means that

$$(\forall \varepsilon > 0)(\exists \delta > 0)(\forall \xi \in S_\delta)(\forall \tau \in I): \|\eta_*(\tau, \xi) - \eta_*(\tau, x_0)\| < \varepsilon \tag{2.9}$$

Selecting ε to be sufficiently small in (2.9), we obtain by using (2.5)

$$g(\eta_*(\tau, \xi)) \geqslant \beta/2 > 0$$

for $\tau \in I$, $\xi \in S_\delta$. Therefore, the mapping $\Pi_*: \tau \to t$ according to the rule

$$t = \Pi_*(\tau, \xi) = \int_0^\tau g(\eta_*(\theta, \xi))\, d\theta$$

is a homeomorphism of I in I.

Direct verification shows that the function

$$x = \eta_*(\Pi_*^{-1}(t, \xi), \xi) \equiv \eta(t, \xi) \tag{2.10}$$

is a solution of system (2.2) that becomes ξ at $t = 0$. The last equation in (2.10) follows from the uniqueness theorem and is equivalent to the following

$$\eta_*(\tau, \xi) \equiv \eta(\Pi_*(\tau, \xi), \xi), \quad \xi \in S_\delta \tag{2.11}$$

Relationships (2.9) and (2.11) indicate that the functional Π_* yields a stable parametrization, with respect to (2.3), of the set of solutions (2.1) of system (2.2).

Corollary. If solution (2.8) of system (2.6) is Lyapunov stable, then solution (2.3) of system (2.2) is orbitally stable.

Thus, the fundamental difficulty to reducing on orbital stability investigation to a study of the Lyapunov stability of the solution of a certain auxiliary system consists of finding the suitable function $g: x \to g(x)$.

Remark. Formulation of the orbital stability criterion mentioned in the Corollary to Theorem 1 is close in its idea to the determination of the Zhukovskii "strength of motion" /3/. Zhukovskii chose one of the phase variables of the system under investigation as the auxiliary variable τ.

3. We will now investigate the orbital stability of the solutions (1.3) and (1.4) of (1.1) by the method mentioned.

We introduce the Suslov variable /10/ as τ by appending the equation

$$\tau^{\cdot} = r \tag{3.1}$$

to (1.1).

We note that the right-hand side of (3.1) satisfies the condition (2.5) since $r_0(t) \neq 0$ by virtue of (1.5).

Eliminating the variable t from (1.1) and (3.1), we arrive at the system

$$2\frac{dp}{d\tau} = q,\quad 2\frac{dq}{d\tau} = -p - \frac{\gamma''}{r},\quad \frac{dr}{d\tau} = \frac{\gamma'}{r} \tag{3.2}$$
$$\frac{d\gamma}{d\tau} = -\frac{q}{r}\gamma'' + \gamma',\quad \frac{d\gamma'}{d\tau} = -\gamma + \frac{p}{r}\gamma'',\quad \frac{d\gamma''}{d\tau} = \frac{q\gamma - p\gamma'}{r}$$

These equations allow of five first integrals /10/, three of which agree with the first three relationships in (1.2) and the remaining two have the form (A and B are arbitrary constants)

$$\begin{aligned}(p^2 - q^2 + \gamma)\cos\tau - (2pq + \gamma')\sin\tau &= A\\ (p^2 - q^2 + \gamma)\sin\tau + (2pq + \gamma')\cos\tau &= B\end{aligned} \tag{3.3}$$

The following 2π-periodic solutions of the system (3.2) correspond to the solutions (1.3) and (1.4),

$$\begin{aligned}&p = p_0,\ q = 0,\ r = r_0(\tau) = [2k\cos\tau +\\ &\quad p_0^{-2}(1 - k^2 - p_0^4)]^{1/2},\ \gamma = \gamma_0(\tau) = k\cos\tau - p_0^2\\ &\gamma' = \gamma_0'(\tau) = -k\sin\tau,\ \gamma'' = \gamma_0''(\tau) = -p_0 r_0(\tau)\end{aligned} \tag{3.4}$$

In conformity with Sect.2, we study the Lyapunov stability of the solution (3.4) of Eq. (3.2) to investigage the orbital stability of the solutions (1.3) and (1.4). We will seek the stability condition of the solution (3.4) by the method of Lyapunov functions. To do this, setting

$$\begin{aligned}&p = p_0 + p_1,\ q = q_1,\ r = r_0(\tau) + r_1,\ \gamma = \gamma_0(\tau) + \gamma_1\\ &\gamma' = \gamma_0'(\tau) + \gamma_1',\ \gamma'' = \gamma_0''(\tau) + \gamma_1''\end{aligned}$$

and omitting the subscript in the perturbations, we write down the first integrals of the perturbed motion equations

$$\begin{aligned}W_1 &= 4p_0p + 2r_0r - 2\gamma + 2p^2 + 2q^2 + r^2\\ W_2 &= 2\gamma_0p + 2\gamma_0'q + \gamma_0''r + 2p_0\gamma +\\ &\quad r_0\gamma'' + 2p\gamma + 2q\gamma' + r\gamma''\\ W_3 &= 2\gamma_0\gamma + 2\gamma_0'\gamma' + 2\gamma_0''\gamma'' + \gamma^2 + \gamma'^2 + \gamma''^2\\ W_4 &= 2p_0\cos\tau p - 2p_0\sin\tau q + \gamma\cos\tau -\\ &\quad \gamma'\sin\tau + (p^2 - q^2)\cos\tau = 2p\,q\sin\tau\\ W_5 &= 2p_0\sin\tau p + 2p_0\cos\tau q + \gamma\sin\tau +\\ &\quad \gamma'\cos\tau + (p^2 - q^2)\sin\tau + 2pq\cos\tau\end{aligned} \tag{3.5}$$

We note that all the integrals (3.5) depend on the variable τ that plays the part of time. This makes necessary the extension of the Pozharitskii /12/ application of the Chetayev method /5/ of constructing the Lyapunov function to the case of a time-dependence of the first integrals of the perturbed motion equations.

We will introduce some notation. We consider functions twice continuously differentiable with respect to x and continuous in t

$$\begin{aligned}&V_i(x, t) = V_i^{(1)}(x, t) + V_i^{(2)}(x, t) + o(\|x\|^2)\\ &(x \in R^n,\ t \in R;\ i = 1, \ldots, m)\end{aligned}$$

Here $V_i^{(1)}\,(V_i^{(2)})$ are terms, linear (quadratic) in x and T and periodic in t, of the Taylor series expansion of $V_i(x, t)$ in the neighbourhood of $x = 0$; $o(\|x\|^2)/\|x\|^2 \to 0$ as $\|x\| \to 0$ uniformly in $t \geqslant t_0$.

Then the following proposition that extends the Rizzito theorem /13/ to the case of non-autonomous first integrals holds.

Theorem 2. For the existence of twice continuously differentiable functions $\psi: R^m \to R$ such that $\psi(V_1, \ldots, V_m)$ and its Hess matrix evaluated for $x = 0$ are positive definite, it is necessary and sufficient that for certain real numbers $\lambda_1, \ldots, \lambda_m$ the following conditions be satisfied:

1) $\sum_{i=1}^{m} \lambda_i V_i^{(1)} \equiv 0$

2) If $\sum_{i=1}^{m} [V_i^{(1)}]^2 = 0$ and $x \neq 0$, there $\sum_{i=1}^{m} \lambda_i V_i^{(2)} > 0$

The proof of this theorem reproduces the well-known reasoning in /13/ pp.125-128 without substantial changes.

We apply Theorem 2 to construct the Lyapunov function from the integrals (3.5). The integral connective that does not contain linear components is

$$\begin{aligned} W = & -p_0^2W_1 - 2p_0W_2 - W_3 + 2kW_4 = \\ & -\{(2p_0^2 - 2k\cos\tau)\,p^2 + (2p_0^2 + 2k\cos\tau)\,q^2 + \\ & 4k\sin\tau pq + 4p_0p\gamma + 4p_0q\gamma' + \gamma^2 + \gamma'^2 + (p_0r + \gamma'')^2\} \end{aligned} \tag{3.6}$$

Let us examine the form (3.6) in a manifold given by the equalities

$$\begin{aligned} & W_1^{(1)} = 4p_0p + 2r_0r - 2\gamma = 0 \\ & W_2^{(1)} = 2p\gamma_0 + 2q\gamma_0' + r\gamma_0'' + 2p_0\gamma + r_0\gamma'' = 0 \\ & W_3^{(1)} = \gamma_0\gamma + \gamma_0'\gamma' + \gamma_0''\gamma'' = 0 \\ & W_4^{(1)} = 2p_0\cos\tau p - 2p_0\sin\tau q + \gamma\cos\tau - \gamma'\sin\tau = 0 \\ & W_5^{(1)} = 2p_0\sin\tau p + 2p_0\cos\tau q + \gamma\sin\tau + \gamma'\cos\tau = 0 \end{aligned} \tag{3.7}$$

The relationships (3.7) enable as to express $\gamma, \gamma', \gamma'', r$ in terms of p and q:

$$\begin{aligned} & \gamma = -2p_0p, \quad \gamma' = -2p_0q, \quad \gamma'' = -\frac{2}{r_0}(\gamma_0p + \gamma_0'q), \\ & r = -\frac{4p_0}{r_0}p \end{aligned} \tag{3.8}$$

Using (3.8), we find the form (3.6) in the manifold (3.7)

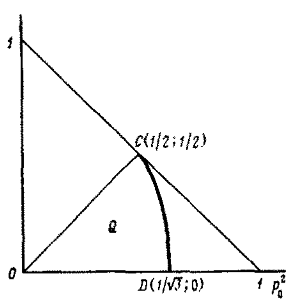
$$\begin{aligned} & W_*(p, q) = \frac{2}{p_0^2r_0^2}(a_{11}p^2 + 2a_{12}pq + a_{22}q^2) \\ & a_{11}(\tau) = (k\cos\tau + p_0^2)(1 - k^2 - 3p_0^4) \\ & a_{12}(\tau) = -k\sin\tau\,(1 - k^2 - 3p_0^4) \\ & a_{22}(\tau) = p_0^2\,(1 - 3k^2 - p_0^4) - k\,(1 - k^2 - 3p_0^4)\cos\tau \end{aligned} \tag{3.9}$$

The necessary and sufficient conditions of positive definiteness of the quadratic form (3.9) with coefficients periodic in τ are sought by using the Silvester criterion and the inequality (1.5). They have the form

$$0 \leqslant k < p_0^2, \ k^2 + 3p_0^4 < 1 \tag{3.10}$$

According to Theorem 2 a sign-definite connective can be constructed from the integrals (3.5) by the Chetayev method if the parameters k and p_0 satisfy inequalities (3.10). The periodic motion (3.4) dependent on k and p_0 for system (3.2) is Lyapunov stable. By virtue of the Corollary from Theorem 1, the inequalities (3.10) yield sufficient conditions for the orbital stability of the solutions (1.3) and (1.4) of system (1.1).

Conditions (3.10) determine a domain Q in the plane Okp_0^2 of the parameters, whose every point corresponds to the orbital stability of the solution (1.3), (1.4) of the equations of motion of a Kovalevskaya gyroscope in the case of Bobylev-Steklov integrability (see the figure).



Remark. 1°. The solution (1.3), (1.4) being studied is stationary in the components p and q of the angular velocity. Consequently, the Lyapunov stability of this solution with respect to the variable p and q in the sense of the definition from /14/ follows from the orbital stability conditions (3.10).

2°. In the Delone case /10/, when $k = 0$, uniform rotations of a Kovalevskaya gyroscope around axes that do not coincide with the principal axis will correspond to the solutions (1.3), (1.4) of the system (1.1). The necessary and sufficient conditions for Lyapunov stability of these motions

$$0 < p_0^2 < 1/\sqrt{3} \tag{3.11}$$

are set up in /15, 16/. For $k = 0$ the inequalities (3.10) also yield the conditions (3.11) to which the segment OD in the figure corresponds.

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ON THE TRANSITION MODE CHARACTERIZING THE TRIGGERING OF A VIBRATOR IN THE SUBSONIC BOUNDARY LAYER ON A PLATE*

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The problem of the development of two-dimensional linear perturbations in a boundary layer, generated by the triggering of a vibrator, is considered. Fourier transformations in the longitudinal coordinate and a Laplace transform in time are used to construct the solution. The inverse transforms are evaluated for large values of the characteristic time t and all values of the longitudinal coordinate x. Domains located downstream of the vibrator are studied in the first of which the perturbations will have the form of Tollmien-Schlichting waves that go over into a wave packet in the second domain. The identity in the structure of the wave packets, which are orthonormalized to the maximum amplitude for this packet for different frequencies of vibrator oscillation is noted.

Vibrating tapes located either on a streamlined surface or within the stream are often used in experimental installations for investigating boundary layer stability. Measurements are made when the harmonic mode of vibrator operation is built up, the transient that originates when it is triggered is considered to be of slight interest and for this reason is not considered. If the frequency of the forced oscillations exceeds the critical value, the formulation of the appropriate boundary value problem is fraught with serious difficulties since the solution must be sought in a class of functions with exponential growth in the longitudinal coordinate. Conditions which ensure the uniqueness of the solution are spoiled since an exponentially increasing eigenfunction of the homogeneous boundary value problem can be appended with arbitrary weight to any solution. The emergence from the situation created relies on the postulate proposed in /1/ according to which the solution at each fixed time

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